Different ranking approaches defining association and agreement measures of paired ordinal data

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Subjective ratings

Observer rating, clinical judgements

level of impairment, severity of disease
diagnostic certainty, clinical status
social functioning, activity of daily life

Self-assessment

QoL, quality of health, sleep disturbance
stress, anxiety

Generating ordinal data

Verbal descriptive scales (VDS)

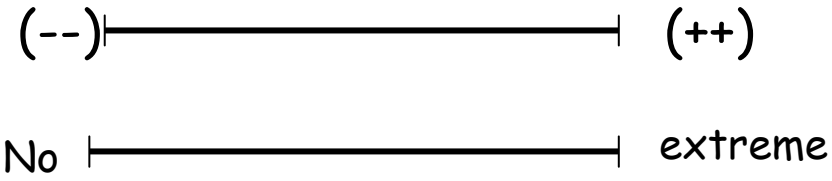
How much….?
[ ] Extremely high
[ ] Very high
[ ] Moderate
[ ] Slight
[ ] Very low
[ ] non-existing

How often….?
[ ] None of the time
[ ] A little of the time
[ ] Some of the time
[ ] Most of the time
[ ] All of the time

Numeric Rating Scales (NRS)

0 1 2 3 4

Visual Analogue Scales (VAS)



Pictogram

- Hierarchical Conditional Scales (Staircases)
- Categorised Continuous Quantitative Data

Rank-invariant properties of ordinaldata

Ordered categorical data are **invariant** under all order-preserving transformations

Non-numbers: Any set of ordered symbols can be used (1,2,3,.., a,b,c…, +, ++, +++, ..)

Change in labelling must not affect the result of data analysis

NO ARITHMETICS (Sums, differences).

AIM

To compare the applications of two ranking approaches to paired ordinal data

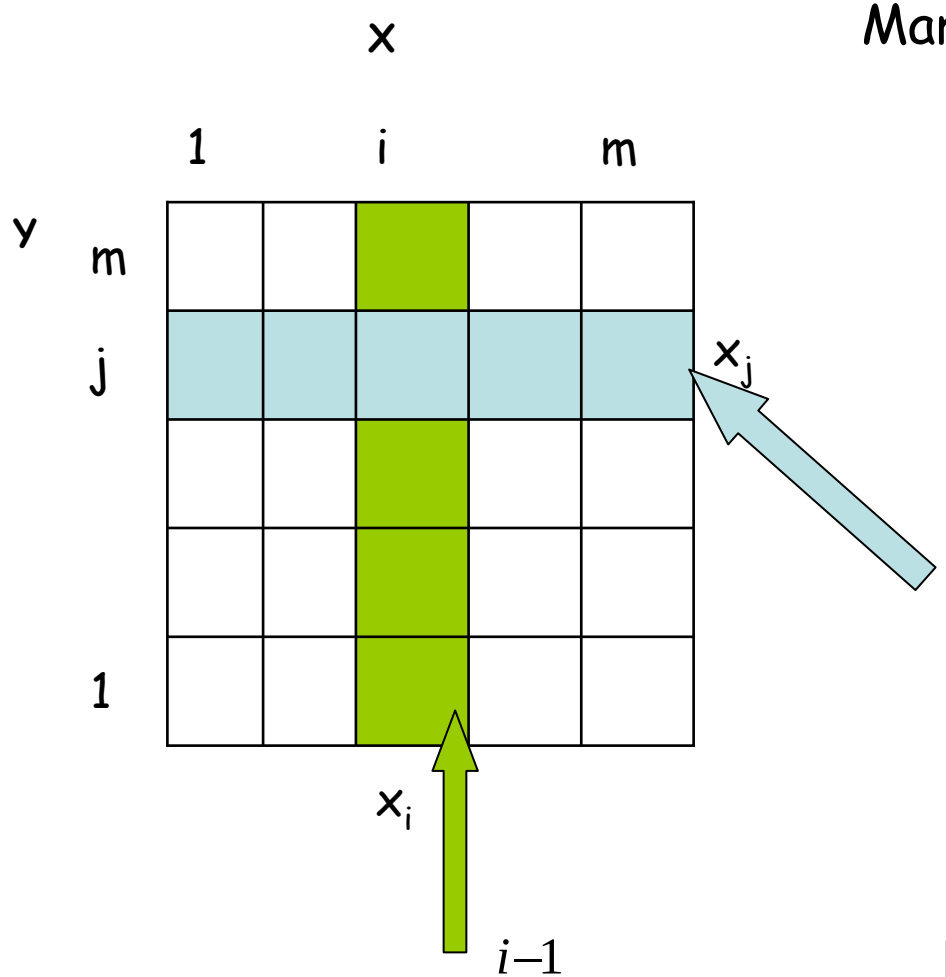
Association:

- variables X: $\{1,\ldots,i,\ldots,m_1\}$, Y:$\{1,\ldots,j,\ldots,m_2\}$

Agreement:

-assessments X:$\{1,\ldots,i,\ldots,m\}$, Y:$\{1,\ldots,j,\ldots,m\}$

Classical ranking, paired data



Marginal distributions $\{x_i\}$, $\{x_j\}$,

i, j: 1,…,m

Ranks tied to each marginal

$$\bar{r}_j = \sum_{\nu=1}^{j-1} x_\nu + \tfrac{1}{2}(x_j + 1)$$

$$\bar{r}_i = \sum_{\nu=1}^{i-1} x_\nu + \tfrac{1}{2}(x_i + 1)$$

Association

The Spearman rank-order correlation coefficient – a measure of **association** between variables

based on rank-differences

adjusted for ties $t^{(X)}$, $t^{(Y)}$

$$d_\nu = r_\nu^{(X)} - r_\nu^{(Y)}$$

$$r_s = \frac{n^3 - n - 6\sum_{\nu=1}^{m} d_\nu^2 - \tfrac{1}{2}(t^{(X)} + t^{(Y)})}{\sqrt{\left(n^3 - n\right)^2 - (t^{(X)} + t^{(Y)})(n^3 - n) + t^{(X)}t^{(Y)}}}$$

without ties:

$$r_s = 1 - \frac{6\sum_{k=1}^{n} d_k^2}{n^3 - n} \qquad k=1, \ldots, n$$

Relationship between perceived back pain and physical health?

		How severe back **pain** have you had during the past four weeks?					
		None	Negli-gible	Mode-rate	Rather severe	Very severe	Tot
Regarding your back pain, how is your per-ceived **physical health?**	Very bad		**1**	**2**	**6**	**4**	13
	Rather bad		**5**	**13**	**24**	**3**	45
	Neither ..nor		**2**	**12**	**8**	**2**	24
	Rather good	**1**		**10**	**6**	**1**	18 **rank 10.5**
	Very good			**1**			1
	Total	1	8	38 **Rank 28.5**	44	10	101

r_s 0.24

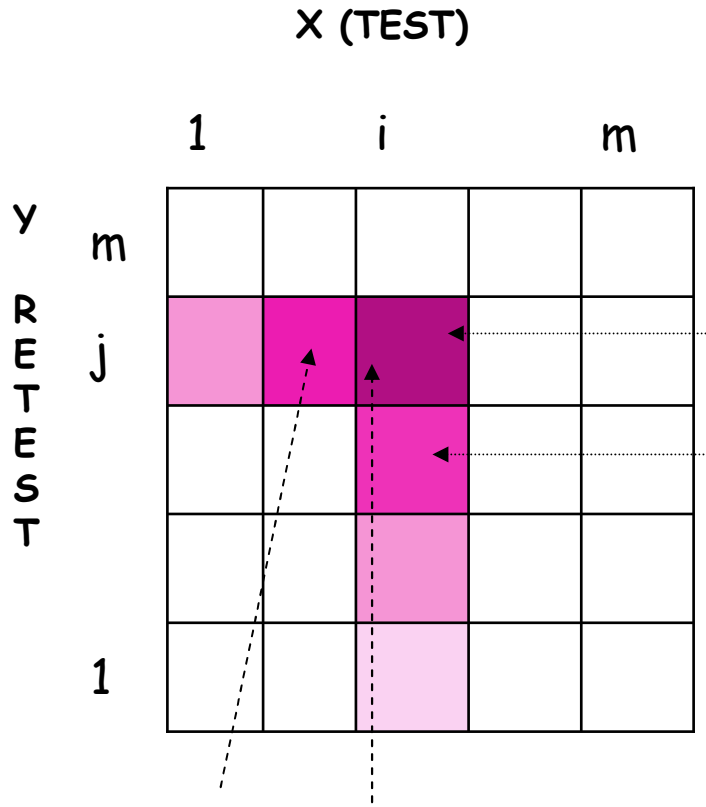
Agreement

RELIABILITY:
The extent to which repeated assessments yield the same result ➔

AGREEMENT
Inter- rater agreement
intra-rater agreement
stability

Augmented ranks based on information from pairs of assessments

The augmented ranking of paired ordinal data



Aug-rank: The ranks are tied to the cells

$$\overline{R}^{(X)}_{i(j-1)} < \overline{R}^{(X)}_{ij}$$

$$\overline{R}^{(Y)}_{(i-1)j} < \overline{R}^{(Y)}_{ij}$$

Agreement

(Svensson) Method based on aug-ranks

Developed for paired ordinal data from all types of scales (incl VAS)

Identify and measure the components of

- **systematic** and
- **individual**

sources of disagreement

Intra-rater agreement in test-retest assessments of perceived back pain

How severe low back pain have you had during the last four weeks?

Percentage Agreement (PA)

78/101= 77%

		First occasion: None	Negligible	Moderate	Rather severe	Very severe	
Second occasion	Very severe				3	3	6
	Rather severe			5	36	7	48
	Moderate		2	33	5		40
	Negligible		5				5
	None	1	1				2
		1	8	38	44	10	101

The pairs of aug-ranks

X: First occasion

Y	None	Negli-gible	Mode-rate	Rather severe	Very severe
Very severe				3 **(90;97)**	3 **(100;100)**
Rather severe			5 **(45;50)**	36 **(70.5;70.5)**	7 **(95;92)**
Mode-rate		2 **(8.5;8.5)**	33 **(26;26)**	5 **(50;45)**	
Negli-gible		5 **(5;5)**			
None	1 **(1;1)**	1 **(2;2)**			

$$\overline{R}_{21} < \overline{R}_{22} < \overline{R}_{23}$$

The pairs of aug-ranks

X: First occasion

Y	None	Negli-gible	Mode-rate	Rather severe	Very severe
Very severe				3 **(90;97)**	3 **(100;100)**
Rather severe			5 **(45;50)**	36 **(70.5;70.5)**	7 **(95;92)**
Mode-rate		2 **(8.5;8.5)**	33 **(26;26)**	5 **(50;45)**	
Negli-gible		5 **(5;5)**			
None	1 **(1;1)**	1 **(2;2)**			

$$\overline{R}_{ij}^{(X)} \neq \overline{R}_{ij}^{(Y)}$$

Dispersion from the Rank-Transformable Pattern of Agreement (RTPA)

The rank transformable pattern of agreement, RTPA

- pairing off the two marginal distributions
- the expected pattern when systematic disagreement only

$$\overline{R}_{ij}^{(X)} = \overline{R}_{ij}^{(Y)}$$

Y \ X	None	Negli-gible	Mode-rate	Rather severe	Very severe	Total
Very severe					6	6
Rather severe				44	4	48
Moderate		2	38			40
Negligible		5				5
None	1	1				2
Total	1	8	38	44	10	101

The Relative Rank Variance, RV, the measure of

- dispersed pairs from the RTPA
- individual/occasional disagreement ("noise")
- Heterogeneity of the group

Y \ X	None	Negli-gible	Mode-rate	Rather severe	Very severe
Very severe				3 **(90;97)**	3 **(100;100)**
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Mode-rate		2 **(8.5;8.5)**	33 **(26;26)**	5 **(50;45)**	
Negli-gible		5 **(5;5)**			
None	1 **(1;1)**	1 **(2;2)**			

$$\Delta\overline{R}_{ij} = \overline{R}_{ij}^{(X)} - \overline{R}_{ij}^{(Y)}$$

$$RV = \frac{6}{n^3}\sum_{i=1}^{m}\sum_{j=1}^{m} x_{ij}\Delta\overline{R}_{ij}^{2}$$

RV=0.003

The Aug-rank coefficient, r_a,

a measure of

- the closeness of pairs to the RTPA
- agreement in aug-ranks ➔ homogeneity

$$r_a = 1 - \frac{n^3 RV}{(n^3 - n) - n^3 IV}$$

Adjusted for ties (internal variance) IV

Conclusions PA: 77%

r_a: 0.997 negligible individual disagreement

Homogeneous group

RTPA reveals slight systematic disagreement,

RP: -0.013, RC: 0.064

y \ X	None	Negli-gible	Mode-rate	Rather severe	Very severe
Very severe				3	3
Rather severe			5	36	7
Mode-rate		2	33	5	
Negli-gible		5			
None	1	1			

r_a

Member of correlation family
Correlation of aug-ranks

Svensson r_a:

$$r_a = \frac{(n^3 - n) - n^3 IV - 6\sum_{i=1}^{m}\sum_{j=1}^{m}(x_{ij} \cdot \Delta\bar{R}_{ij}^2)}{\sqrt{(n^3 - n)^2 - 2n^3(n^3 - n)IV + \left(n^3 \cdot IV\right)^2}}$$

Spearman r_s

$$r_s = \frac{n^3 - n - 6\sum_{v=1}^{m} d_v^2 - \tfrac{1}{2}(t^{(X)} + t^{(Y)})}{\sqrt{\left(n^3 - n\right)^2 - (t^{(X)} + t^{(Y)})(n^3 - n) + t^{(X)}t^{(Y)}}}$$

Different applications, interpretations

Association IS NOT Agreement

r and r_s

are sometimes used

as reliability measures, but

This is WRONG

The aug-rank methods are

designed for

separate analysis of the

individual and systematic

components of (dis)agreement

Y \ X	A	B	C	D	total
D				3	3
C				31	31
B			12		12
A	2	11			13
total	2	11	12	34	59

PA, 8%
The RTPA:
r_a, 1

Systematic d-a
RP, -0.53; RC, 0.36

r_s, 0.97 κ, -0.12

Y \ X	A	B	C	D	tot
D		1	3	30	34
C	1	1	7	3	12
B	1	6	1	1	9
A	2	1	1		4
	4	9	12	34	59

PA, 76%
Individual d-a:
r_a, 0.78

Marginal homogeneity
RP=RC=0

r_s, 0.77; κ, 0.60

Y \ X	A	B	C	D	tot
D			2		2
C		1		27	28
B	1	1	8	5	15
A	1	9	3	1	14
	2	11	13	33	59

PA, 3%
Individual d-a:
r_a, 0.90

Systematic d-a:
RP, -0.56; RC, 0.35

r_s, 0.73; κ, -0.18

Some references

- www.oru.se/esi/svensson

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