Power and Log-linear non-uniform association models in agreement studies

Fabien Valet[1] and Jean-Yves Mary[2]

[1] Institut Curie, Ecole des Mines de Paris, INSERM U900, Paris, FRANCE
[2] INSERM U717, Hôpital Saint-Louis, Paris, FRANCE

Ordinal rating scales (ORS)

- ORS are often used in biomedical fields to measure clinical outcomes
- When objects are rated twice using an ORS, ratings may vary
- Category distinguishability is a major component of the reproducibility of their ratings
- Log-linear non-uniform association models (NUA) are able to detect and to test patterns of distinguishability
- Objective: to investigate the ability of reproducibility designs to detect significant heterogeneity among distinguishabilities using these models

Method: log-linear models (1)

Contingency table for ORS with 5 grades

		B				
		1	**2**	**3**	**4**	**5**
A	**1**	n_{11}	n_{12}	n_{13}	n_{14}	n_{15}
	2	n_{21}	n_{22}	n_{13}	n_{14}	n_{15}
	3	n_{31}	n_{32}	n_{33}	n_{34}	n_{35}
	4	n_{41}	n_{42}	n_{43}	n_{44}	n_{45}
	5	n_{51}	n_{52}	n_{53}	n_{54}	n_{55}

n_{ij} is the number of objects rated i by A and j by B

$n_{ij} \sim P(m_{ij})$ $\quad$ $m_{ij} = E(n_{ij})$

Independence

$$Log(m_{ij}) = \mu + \lambda_i^A + \lambda_j^B$$

Uniform Association *(Goodman, 1979)*

$$Log(m_{ij}) = \mu + \lambda_i^A + \lambda_j^B - \frac{1}{2}(i-j)^2 \times \beta$$

parameters interpretation

OR

$$\tau_{ij} = e^{\beta(i-j)^2}$$

DD *(Darroch et al.,1986)*

$$\gamma_{ij} = 1 - e^{-\beta(i-j)^2}$$

Method: log-linear models (2)

Non-Uniform Association (NUA)

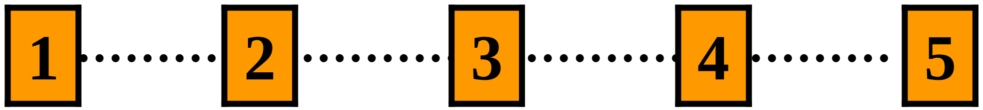
$$Log(m_{ij}) = \mu + \lambda_i^A + \lambda_j^B - \frac{|i-j|}{2} \times \sum_{k=\min(i,j)}^{\max(i,j)-1} \beta_{k,k+1}$$

$$\gamma_{ij} = 1 - e^{\left\{-|i-j| \times \sum_{k=\min(i,j)}^{\max(i,j)-1} \beta_{k,k+1}\right\}}$$

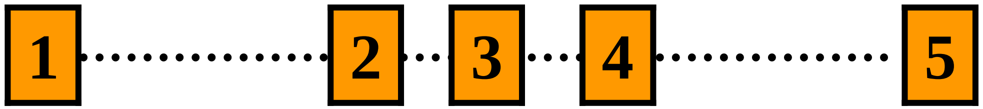
(Valet et al. 2007)

Typical examples of distinguishability patterns

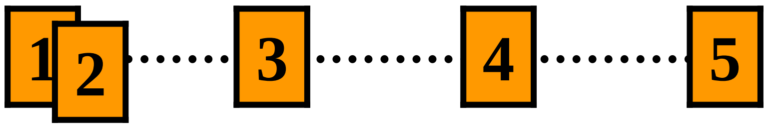
a. $\gamma_{1,2} = \gamma_{2,3} = \gamma_{3,4} = \gamma_{4,5}$



b. $\gamma_{1,2} = \gamma_{4,5} > \gamma_{2,3} = \gamma_{3,4}$



c. $\gamma_{1,2} << \gamma_{2,3} = \gamma_{3,4} = \gamma_{4,5}$



Method: power estimation in NUA models

- Simulation of Full Multinomial distributions

  - $n_{ij} \sim M(\pi_{ij}, N)$ where $\pi_{ij} = f(\mu, \lambda_i^A, \lambda_j^B, \beta_{k,k+1})$

  - We assume
    - marginal homogeneity between readers
    - homogeneous marginal distribution within reader

  - $\beta_{k,k+1}$ fixed → non-linear system of 4 equations with 4 unknown parameters (DFSANE, LaCruz et al. 2006)

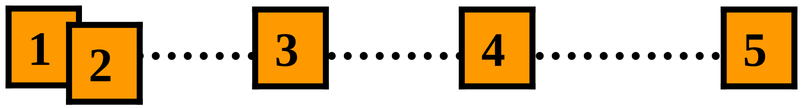
Method: power estimation in NUA models

- Simulated association patterns: definition of H_0 and H_1

$$H_0 : \beta_{12} = \beta_{23} = \beta_{34} = \beta_{45}$$

$$H_1 : \beta_{12} = \beta_{45} > \beta_{23} = \beta_{34}$$

$$H_1 : \beta_{12} << \beta_{23} = \beta_{34} = \beta_{45}$$



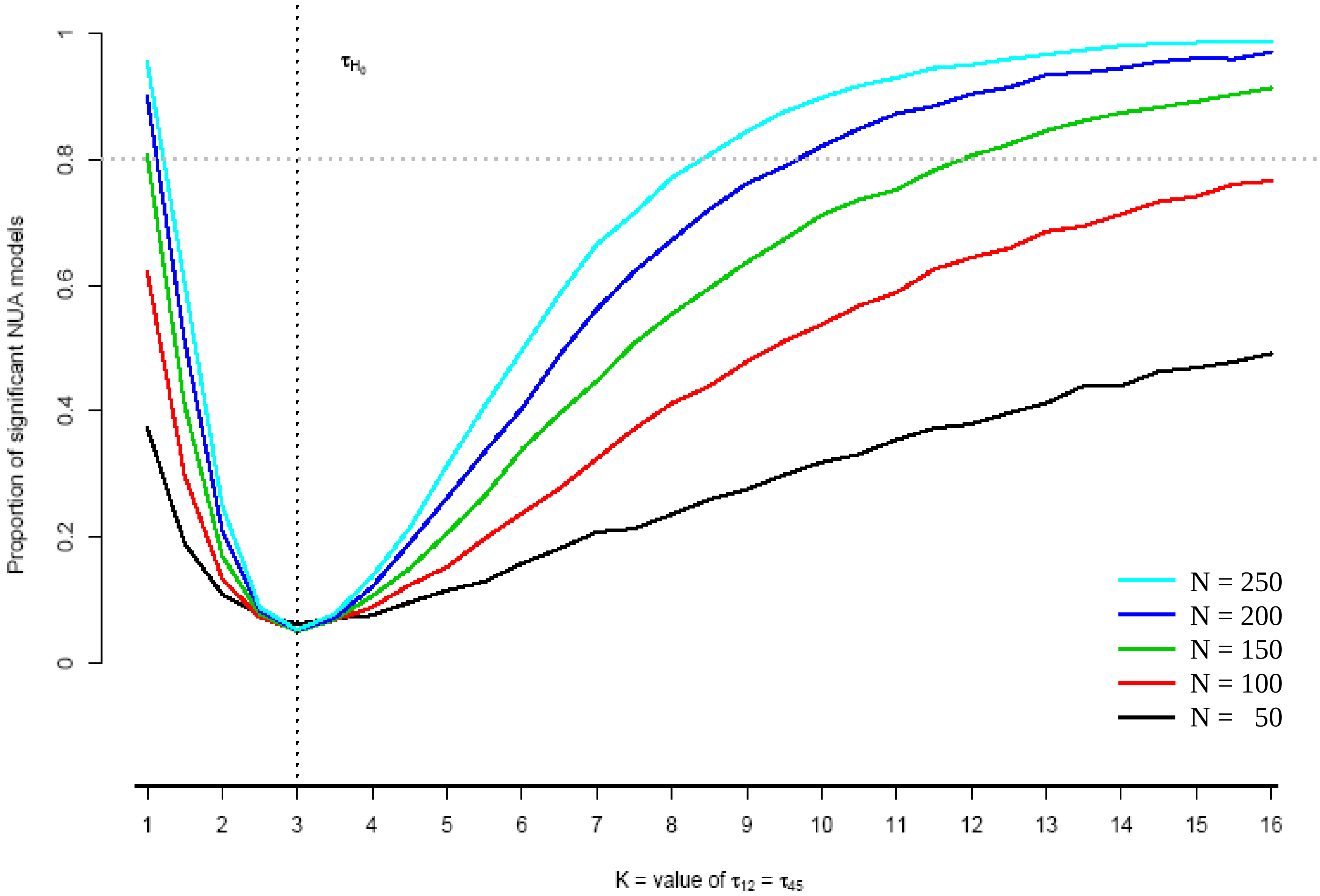
- Simulations = 10000

  proportion of significant NUA models under H_1 (Power) and H_0 (Type I error)

Results (1)

$H_0 : \tau_{12} = \tau_{23} = \tau_{34} = \tau_{45} = 3$

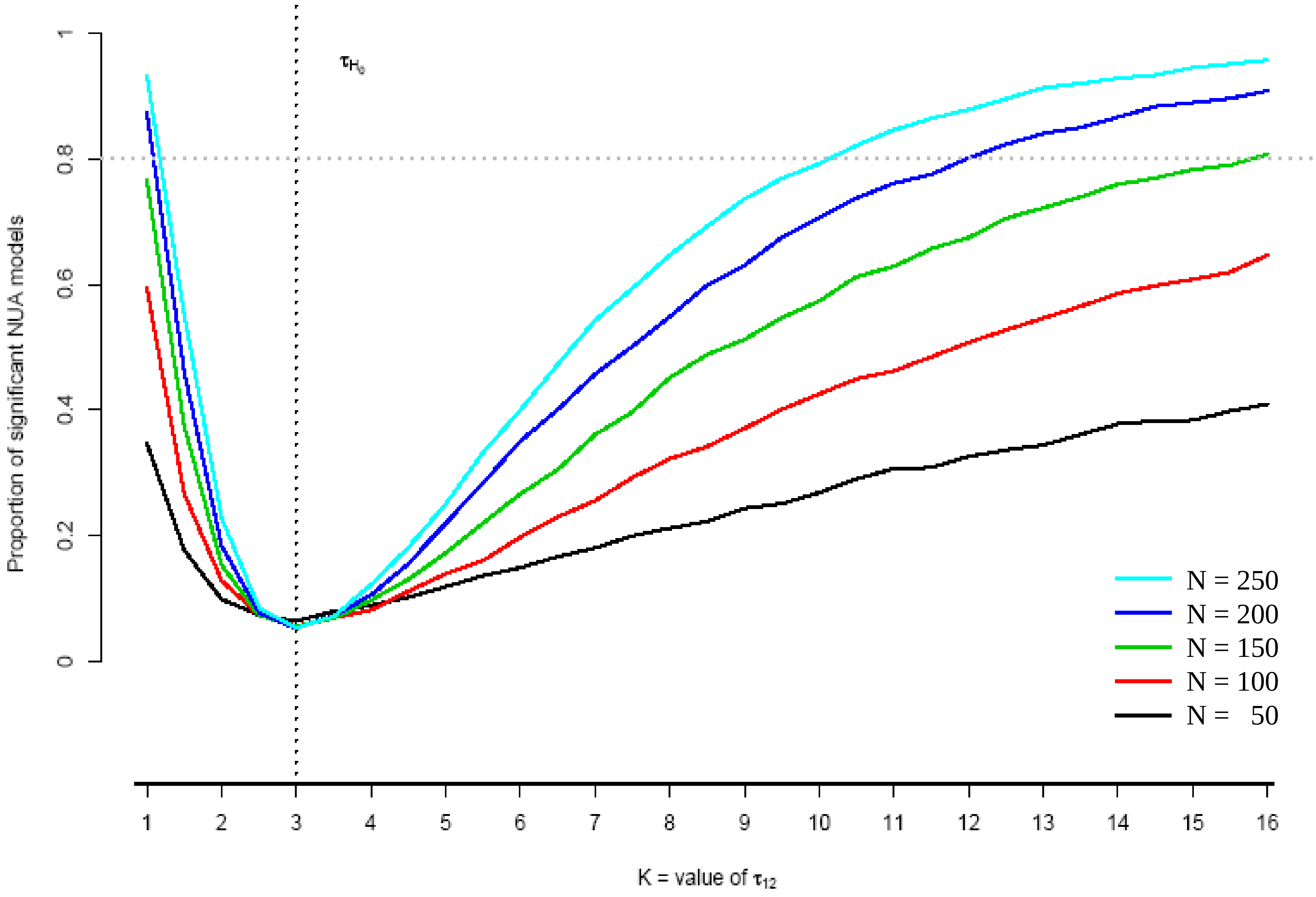
$H_1 : \tau_{12} = \tau_{45} = K$



Results (2)

$H_0 : \tau_{12} = \tau_{23} = \tau_{34} = \tau_{45} = 3$

$H_1 : \tau_{12} = K$



Results : power as a function of N and K

$$H_0 : \tau_{12} = \tau_{23} = \tau_{34} = \tau_{45} = 3$$

	$H_1 : \tau_{12} = \tau_{45} = K$					$H_1 : \tau_{12} = K$				
N=	***50***	***100***	***150***	***200***	***250***	***50***	***100***	***150***	***200***	***250***
τ_K										
1	.37	.62	.81	.90	.95	.35	.59	.77	.87	.93
1.5	.19	.30	.41	.51	.60	.18	.27	.37	.46	.55
2	.11	.13	.17	.21	.25	.10	.13	.15	.18	.23
2.5	.08	.07	.08	.08	.09	.07	.07	.07	.08	.08
3	.06	.05	.05	.05	.05	.06	.05	.05	.05	.05
4	.07	.09	.11	.12	.14	.09	.08	.10	.10	.12
8	.24	.41	.55	.67	.77	.21	.32	.45	.55	.64
10	.32	.54	.74	.82	.90	.27	.43	.57	.71	.79
16	.49	.77	.91	.97	.99	.41	.65	.81	.91	.96

Results : power as a function of N and K

$$H_0 : \tau_{12} = \tau_{23} = \tau_{34} = \tau_{45} = 3$$

$$H_1 : \tau_{12} = \tau_{45} = K \qquad\qquad H_1 : \tau_{12} = K$$

	$H_1 : \tau_{12} = \tau_{45} = K$					$H_1 : \tau_{12} = K$				
N=	***50***	***100***	***150***	***200***	***250***	***50***	***100***	***150***	***200***	***250***
τ_K										
1	.37	.62	**.81**	**.90**	**.95**	.35	.59	.77	**.87**	**.93**
1.5	.19	.30	.41	.51	.60	.18	.27	.37	.46	.55
2	.11	.13	.17	.21	.25	.10	.13	.15	.18	.23
2.5	.08	.07	.08	.08	.09	.07	.07	.07	.08	.08
3	.06	.05	.05	.05	.05	.06	.05	.05	.05	.05
4	.07	.09	.11	.12	.14	.09	.08	.10	.10	.12
8	.24	.41	.55	.67	.77	.21	.32	.45	.55	.64
10	.32	.54	.74	**.82**	**.90**	.27	.43	.57	.71	.79
16	.49	.77	**.91**	**.97**	**.99**	.41	.65	**.81**	**.91**	**.96**

Nominal level of Type one error

Power above 80%

Results : power as a function of N and K

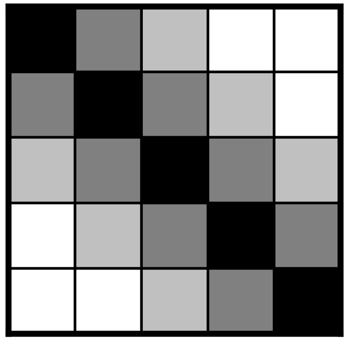
$$H_0 : \tau_{12} = \tau_{23} = \tau_{34} = \tau_{45} = 3$$

$$H_1 : \tau_{12} = \tau_{45} = K \qquad\qquad H_1 : \tau_{12} = K$$

	$H_1 : \tau_{12} = \tau_{45} = K$					$H_1 : \tau_{12} = K$				
N=	***50***	***100***	***150***	***200***	***250***	***50***	***100***	***150***	***200***	***250***
τ_K					**231**					
1	.37	.62	.81	.90	.95	.35	.59	.77	.87	.93
1.5	.19	.30	.41	.51	.60	.18	.27	.37	.46	.55
2	.11	.13	.17	.21	.25	.10	.13	.15	.18	.23
2.5	.08	.07	.08	.08	.09	.07	.07	.07	.08	.08
3	.06	.05	.05	.05	.05	.06	.05	.05	.05	.05
4	.07	.09	.11	.12	.14	.09	.08	.10	.10	.12
8	.24	.41	.55	.67	.77	.21	.32	.45	.55	.64
9				**.75**	**.80 .82**					
10	.32	.54	.74	.82	.90	.27	.43	.57	.71	.79
16	.49	.77	.91	.97	.99	.41	.65	.81	.91	.96

Conclusion → results in a 5 x 5 table

- Simulations done with $\tau_{H0} = 3$ → reasonable assumption in agreement studies



- For $N \geq 200$ NUA models are able to detect (>80%)
  - $\tau_K < 1.25$ → DD < .20
  - $\tau_K > 10$ → DD > .90

Discussion: what's next ?

- How do marginal heterogeneity impact NUA models for heterogeneity of degree of distinguishability detection ?

- Application to I x I contingency tables (I=3,4,6,7)

- R package → loglinear association,
  → sample size calculation for agreement studies

Bibliography

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- **Darroch and McCloud.** Category distinguishability and observer agreement. *Australian Journal of Statistics* 1986; 28:371-388

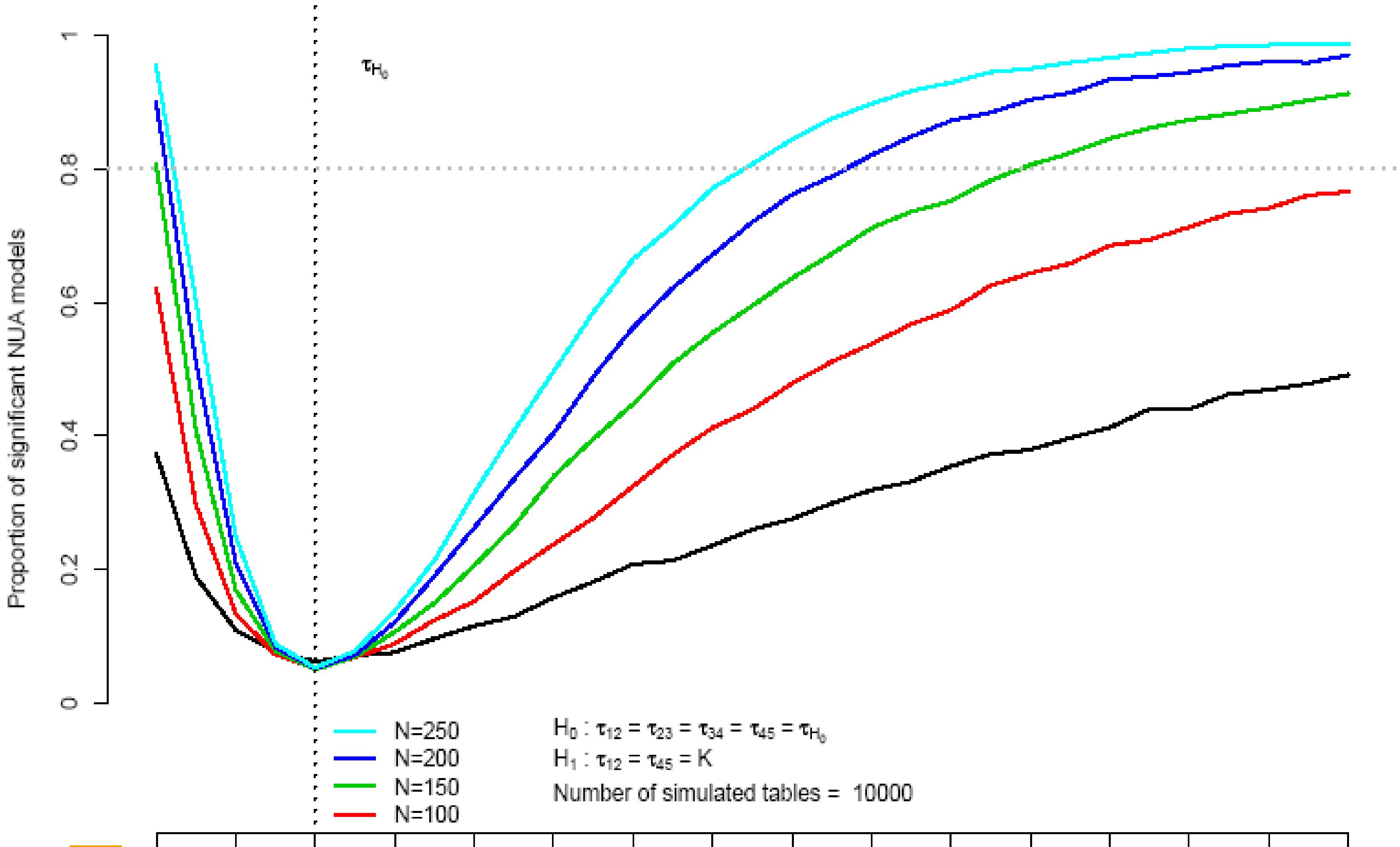
- **W LaCruz, *et al*.** Spectral residual method without gradient information for solving large-scale nonlinear systems of equations. *Mathematics of Computation* 2006; 75, 1429-1448.

APPENDIX

Results (1)

$H_0 : \tau_{12} = \tau_{23} = \tau_{34} = \tau_{45} = 3$

$H_1 : \tau_{12} = \tau_{45} = K$

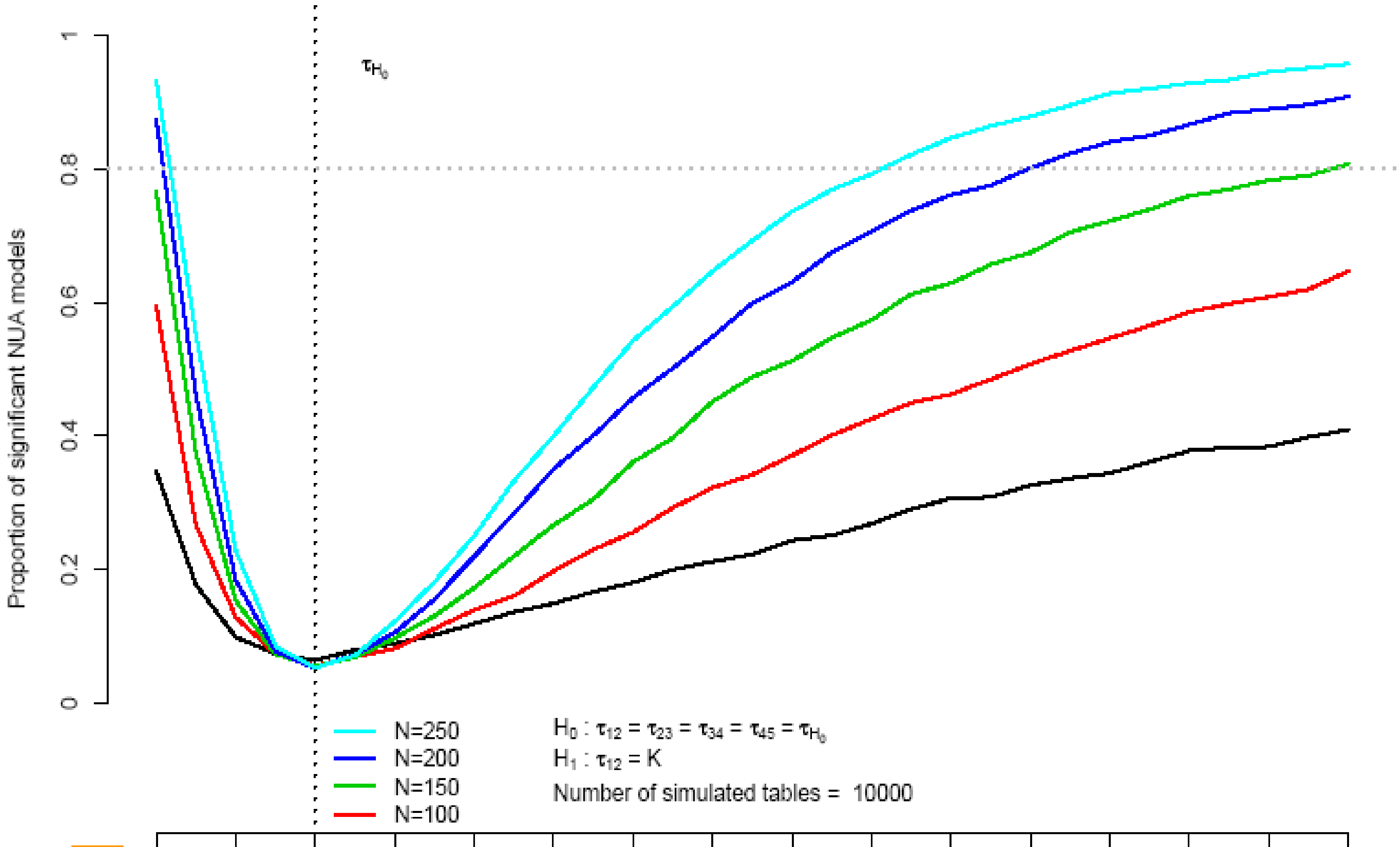


τ_{H1}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
γ_{H1}	.00	.50	.67	.75	.80	.83	.86	.88	.89	.90	.91	.92	.92	.93	.93	.94
β_{H1}	.00	.69	1.10	1.38	1.61	1.79	1.95	2.08	2.20	2.30	2.40	2.48	2.56	2.64	2.71	2.77

Results (2)

$H_0 : \tau_{12} = \tau_{23} = \tau_{34} = \tau_{45} = 3$

$H_1 : \tau_{12} = K$



τ_{H1}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
γ_{H1}	**.00**	**.50**	**.67**	**.75**	**.80**	**.83**	**.86**	**.88**	**.89**	**.90**	**.91**	**.92**	**.92**	**.93**	**.93**	**.94**
β_{H1}	**.00**	**.69**	**1.10**	**1.38**	**1.61**	**1.79**	**1.95**	**2.08**	**2.20**	**2.30**	**2.40**	**2.48**	**2.56**	**2.64**	**2.71**	**2.77**

Method: power estimation in NUA models

Estimation of simulations parameters

	1	2	3	4	5	
1	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	$\boldsymbol{p_{1+}}$
2	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	$\boldsymbol{p_{2+}}$
3	p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	$\boldsymbol{p_{3+}}$
4	p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	$\boldsymbol{p_{4+}}$
5	p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	$\boldsymbol{p_{5+}}$

$$N \times p_{i+} = e^{\mu} \left\{ \sum_{j=1}^{5} e^{\left(\mu + \lambda_i^A + \lambda_j^B - \frac{|i-j|}{2} \times \sum_{k=\min(1,j)}^{\max(1,j)-1} \beta_{k,k+1} \right)} \right\}$$

Marginal Homogeneity

$$\lambda_i^A = \lambda_i^B = \lambda_i$$

Symmetry

$$\lambda_1 = \lambda_5, \; \lambda_2 = \lambda_4$$

$\beta_{k,k+1}$ fixed

$$\begin{cases} p_{i+} = f(\mu, \lambda_1, \lambda_2, \lambda_3) = \dfrac{1}{5} & i = 1,2,3 \\ 2\lambda_1 + 2\lambda_2 + \lambda_3 = 0 \end{cases}$$

non-linear system of 4 equations
with 4 unknown parameters

Method: power estimation in NUA models

- Simulation of a 5 x 5 table with an association pattern

for each set $S = \{\beta_{k,k+1}\}$ k=1, 2, 3, 4

we have a set of solutions $\{\mu, \lambda_1, \lambda_2, \lambda_3\}_S$

$$N \times p_{ij} = e^{\mu + \lambda_i^A + \lambda_j^B - \frac{|i-j|}{2} \times \sum_{k=\min(i,j)}^{\max(i,j)-1} \beta_{k,k+1}}$$

$$p_{ij}, \quad (i, j) \in \{1,2,3,4,5\}^2$$

	1	2	3	4	5
1	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}
2	m_{21}	m_{22}	m_{23}	m_{24}	m_{25}
3	m_{31}	m_{32}	m_{33}	m_{34}	m_{35}
4	m_{41}	m_{42}	m_{43}	m_{44}	m_{45}
5	m_{51}	m_{52}	m_{53}	m_{54}	m_{55}

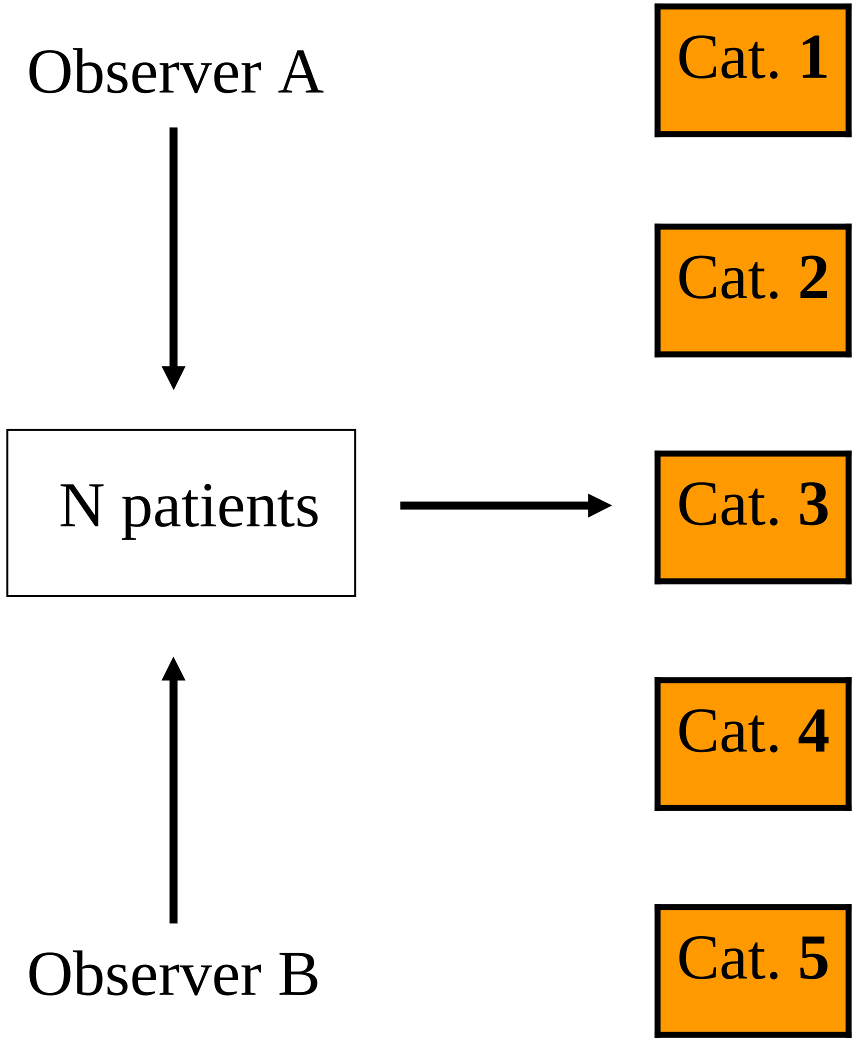
$$Mult(p_{ij}, N)$$

ORS: examples

Intensity of a response		Pathology grading system
Absent	Cat. **1**	Grade cannot be assessed
Mild	Cat. **2**	Well differentiated (low grade)
Moderate	Cat. **3**	Moderately differentiated (intermediate grade)
Important	Cat. **4**	poorly differentiated (high grade)
Severe	Cat. **5**	Undifferentiated (high grade)

ORS: reproducibility and contingency tables

Ratings reproducibility



Contingency table

A

B	1	2	3	4	5
1	n_{11}	n_{12}	n_{13}	n_{14}	n_{15}
2	n_{21}	n_{22}	n_{13}	n_{14}	n_{15}
3	n_{31}	n_{32}	n_{33}	n_{34}	n_{35}
4	n_{41}	n_{42}	n_{43}	n_{44}	n_{45}
5	n_{51}	n_{52}	n_{53}	n_{54}	n_{55}