Estimating the adjusted risk difference in observational studies using propensity score-based weighting

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Overview

• limitation of generalised linear model (GLM) for estimating risk difference

• propensity score-based weighting method

• simulation study design and results
Adjusted risk difference

• risk difference is often the parameter of interest in epidemiologic studies

• binomial GLMs with identity link function (for risk difference) often fail to converge when confounders are adjusted for

• odds ratio scale (GLM with logit link) most natural for confounder-adjusted estimates
Propensity score methods

• propensity score is the probability of being exposed (E) given confounders (X)
  \[ \Pr(E=1 \mid X) \]
• used to balance comparison between exposed and non-exposed
• weighted method has application to risk difference estimation in context of GLMs
  – exposure status is the only covariate
  – fitting algorithm guaranteed to converge
Propensity score-based weighting method

• binomial regression of outcome on exposure (using identity link) weighted by inverse probability of exposure (IPWs)

• propensity scores estimated after fitting logistic model of exposure on confounders

• \( \text{IPW} = \frac{1}{(\text{Probability of observed exposure})} \)

  \[ = \frac{1}{\Pr (E = 1|X)} \quad \text{if } E = 1 \text{ (exposed)} \]

  \[ = \frac{1}{[1 - \Pr (E = 1|X)]} \quad \text{if } E = 0 \text{ (non-exposed)} \]
Aim

Use simulation to estimate the bias and confidence interval coverage of the propensity score-based weighting method for estimating the adjusted risk difference in observational studies.
Simulation model

- exposure status model:
  \[
  \text{logit}(\pi_e) = \alpha_e + \beta_e B + \gamma_e C
  \]

- outcome status model:
  \[
  \text{logit}(\pi_o) = \alpha_o + \beta_o B + \gamma_o C + \delta E
  \]

B – binary confounder, \(B \sim \text{Bernoulli}(0.5)\)
C – continuous confounder, \(C \sim \text{Normal}(0,1)\)
E – exposure (0 for non-exposed; 1 for exposed)
O – outcome (0 for non-diseased; 1 for diseased)
Simulation study design parameters

- design parameters values
  - risk in non-exposed group: 5%, 10%, 15%
  - risk ratio: 1, 2, 3
  - confounder-to-exposure (CE) odds ratio: 1, 2, 5
  - confounder-to-outcome (CO) odds ratio: 1, 2, 5
  - sample size: 100, 250, 500, 1000, 2500, 5000
Simulation design

• 2000 datasets generated for each of the 486 combinations of parameter values

• propensity score-based weighting method applied to each dataset saving the
  – estimated risk difference
  – 95% confidence interval
Simulation: properties estimated

- absolute bias of estimate
  - mean of estimates minus true risk difference
- “empirical” standard error (SE)
  - standard deviation of point estimates
- standardised bias – bias as % of SE
  - (absolute bias/standard error)*100%
- coverage of 95% confidence interval
  - % of intervals that include true risk difference
Standardised bias
non-exposed risk = 5%; exposed risk = 5%

<table>
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Confidence interval coverage versus sample size
non-exposed risk = 5%; exposed risk = 5%

Graphs by confounder-to-exposure odds ratio (CE) and confounder-to-outcome odds ratio (CO)
Kernel density plot of propensity scores for exposed and non-exposed groups.
Summary/Implications

- method generally has low bias, good coverage
- coverage may be compromised when
  - there is a marked separation between groups on the propensity scores and
  - sample size is too small to adequately represent each group at the extremes of propensity score distribution
- method also has application to RCTs
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Estimates and confidence intervals from 25 randomly selected datasets

- risk difference estimate
- 95% confidence interval
Proportion of non-exposed subjects with propensity score >0.8 that have the disease

Fraction proportion with disease

N = 100
N = 250
N = 500
N = 1000
N = 2500
N = 5000
N = 10000
N = 25000
N = 50000